Lesson 3-5: Quick Graphs of Linear Equations

Graphing lines…are there shortcuts?

In the last lesson set we learned (or recalled) how to graph a line. We built a table of ordered pairs (x, y coordinate points), plotted them and connected them.

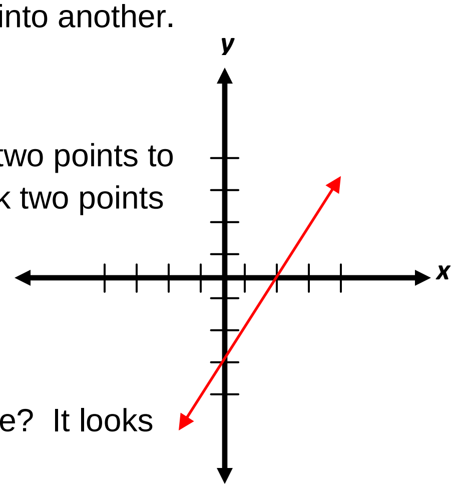
Today, we're going to discover some shortcuts that will make graphing a line much easier and quicker. The key to the shortcuts is being able to recognize different forms a linear equation can take. There are three basic forms:

Form Name	Example	General form	…where…
Standard Form	$2x - 3y = -1$	$Ax + By = C$	A, B and C are real numbers Here $A = 2$, $B = -3$ and $C = -1$
Slope-Intercept Form	$y = 4x - 3$	$y = mx + b$	m is the slope … here $m = 4$ b is the y-intercept … here $b = -3$
Point-Slope Form (next lesson)	$y - 2 = 5(x + 1)$	$y - y_1 = m(x - x_1)$	m is the slope … here $m = 5$ (x_1, y_1) is a point on the line … here the point is (-1, 2)

If you can learn to recognize these line forms, you can quickly pull the critical information from them and graph the line.

It is ***very*** important that you realize you can convert any one form into another.

Pick two, any two…



Recall from geometry that good old Euclid had said we only need two points to make a line. Consider the graph to the right. If you could only pick two points to use in graphing this line, which two would you choose?

I'll bet most you would pick the x- and y-intercepts: where the line crosses the x and y axes. Let's think about those two points.

Looking at the graph, what would you estimate the y-intercept to be? It looks like (0, -3). Let's say it this way: ***when x is 0, then y is -3***.

What about the x-intercept? It looks like (2, 0), or ***when y is 0, then x is 2***.

In other words, to find these two points, try the equation first with $x = 0$, solve for y. Next try the equation with $y = 0$ and solve for x.

Lesson 3-5: Quick Graphs of Linear Equations

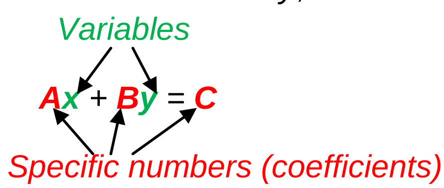
The Standard Form of a linear equation

The ***standard form*** of a linear equation makes it very easy to find the two points where the line crosses the axes. If you recall from the table above, the standard form looks like this: $Ax + By = C$ where *A, B* and *C* are any real numbers. Don't let all the letters confuse you. The variables are *x* and *y*; *A, B* and *C* are just placeholders for actual, specific numbers. The *math* term we use for these numbers is ***coefficients***: a coefficient is the number just before a variable. Here are a couple examples:

$3x - 7y = 13$ The coefficients are $A = 3$, $B = -7$ and $C = 13$

$-\frac{3}{2}x + \frac{7}{9}y = -\frac{11}{2}$ The coefficients are $A = -\frac{3}{2}$, $B = \frac{7}{9}$ and $C = -\frac{11}{2}$

Again, the variables are *x* and *y*; the numbers in the equation are represented by *A, B & C.*



How to graph the standard form

The standard form makes it extremely easy to get the *x*- and *y*-intercepts:

1) Just plug 0 in for *y*, solve for *x* to get the *x*-intercept.

2) Then plug 0 in for *x*, solve for *y* to get the *y*-intercept.

Example – graph 3*x* + 2*y* = 7

This is in standard form. The coefficients are *A = 3, B = 2* and *C=7.* Find the *x*- and *y*-intercepts. Then plot those points and draw a line through them:

***x*-intercept**

$3x + 2(0) = 7$

$3x = 7$

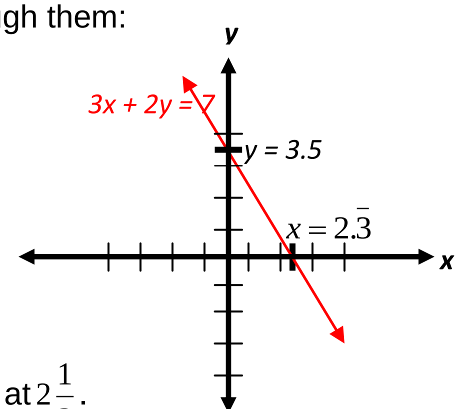
$x = \frac{7}{3} = 2\frac{1}{3} = 2.\bar{3}$

***y*-intercept**

$3(0) + 2y = 7$

$2y = 7$

$y = 3.5$



The line crosses the *y*-axis at $3\frac{1}{2}$ and crosses the *x*-axis at $2\frac{1}{3}$.

For another example, see example 1 on page 78 of the text.

Lesson 3-5: Quick Graphs of Linear Equations

The slope-intercept form

The slope intercept form is $y = mx + b$ where m is the slope of the line and b is the y-intercept. Here are a couple examples:

$y = 7x + 2$ The slope m is 7 and the y-intercept is 2 or (0, 2).

$y = -½x – 3$ The slope m is -½ and the y-intercept is -3 or (0, -3).

To graph a line in slope-intercept form, plot the y-intercept and use the slope to find the 2nd point. Recall that slope is $\frac{rise}{run}$ … *rise* is the change in y and *run* is the change in x.

Also recall that we always go left to right with slope: positive slope goes up and negative slope goes down:

A slope of 7 means up 7 and right 1 (because $7 = \frac{7}{1}$).

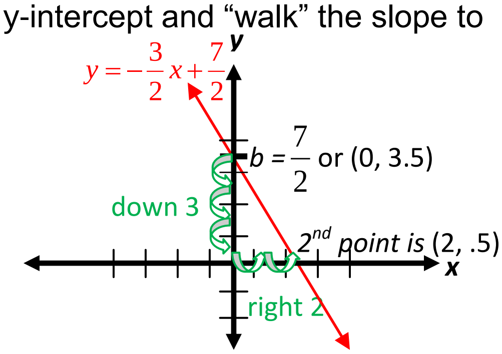
A slope of $-\frac{1}{2}$ means down 1 and right 2.

Example – graph $y = -\frac{3}{2}x + \frac{7}{2}$

This is in slope-intercept form. To graph it, find the y-intercept and "walk" the slope to get the 2nd point:

The y-intercept is $\frac{7}{2}$ or (0, 3.5)

The slope m is $\frac{-3}{2}$ (down 3 and right 2)



Another way of finding the 2nd point is to "add" the slope to the y-intercept point. When we "walk" the slope, really what we are doing is "adding" the *rise* to the y value of the y-intercept and "adding" the *run* to the x value of the y-intercept.

Rise is -3 … the y value of the y-intercept is 3.5: -3 + 3.5 = .5. This is the y value of the 2nd point.

Run is 2 … the x value of the y-intercept is 0: 2 + 0 = 2. This is the x value of the 2nd point.

…the 2nd point is (2, .5)

For another example, see example 2 on page 79 of the text.

All forms are created equal!

The Declaration of Independence states that all people are created equal. Well, the same thing is true about the different forms a linear equation can take.

Take a look at the graph for the standard form equation $3x + 2y = 7$ we worked on first above. Now, compare that graph with the graph of the slope-intercept equation $y = -\frac{3}{2}x + \frac{7}{2}$ we worked with above. They look identical! How can that be? They are different equations right?

Wrong. They are the same equation, just "reformatted." Remember when we worked with ***literal equations***? They are equations with more than 1 variable. We found we could solve a literal equation for either of the variables. In other words, we found that we could manipulate the equation as long as we used our normal algebra rules.

That means that if we have an equation for a line, we can manipulate it into either the standard form or the slope-intercept form. If we have a line in standard form, we can convert it into slope-intercept form, if we want.

Let's try it with our two equations. Let's try to convert the standard form equation into the equivalent slope-intercept equation.

$$3x + 2y = 7 \qquad \text{this is in standard form...}$$

$$\underline{-3x \qquad\quad -3x} \qquad \textit{get } y \textit{ by itself ...subtract } 3x \textit{ from both sides}$$

$$2y = -3x + 7$$

$$\frac{\not{2}}{\not{2}}y = \frac{-3}{2}x + \frac{7}{2} \quad \textit{...divide both sides by } 2$$

$$y = -\frac{3}{2}x + \frac{7}{2} \quad \text{...and now we have the slope-intercept form!}$$

So, we converted the standard form of the equation into its equivalent slope-intercept form. Even though the equations look very different, they are just different forms of the same line.